PG – 851

I Semester M.C.A. (2 Years Course) Degree Examination, August/September 2021
(CBCS Scheme) (2020 – 2021 and Onwards)
COMPUTER SCIENCE
1MCA2 : Discrete Mathematics

Time : 3 Hours Max. Marks : 70

***Instruction** : Answer **any five** from Part – **A** and **any four** from Part – **B**.*

PART – A

Answer **any five** questions. **Each** question carries **six** marks. **(5×6=30)**

1. a) If A, B and C are three sets, then prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. **3**

   b) Prove that "null set is a subset of every set". **3**

2. a) Define one-to-one and on-to functions with example. **3**

   b) Show that $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$. **3**

3. a) Let x be the set of factors of 12 and let $\leq$ be the relation divisor i.e., $x \leq y$, if and only if x divides y. Draw the Hasse diagram of $(x, \leq)$. **3**

   b) Find the values of the extended binomial coefficients $\binom{-2}{3}$ and $\binom{\frac{1}{2}}{3}$. **3**

4. Obtain an explicit form for the following sequence a_n defined recursively by $a_n = 2a_{n-1} + 1$ for $n \geq 2$, with $a_1 = 3$. **6**

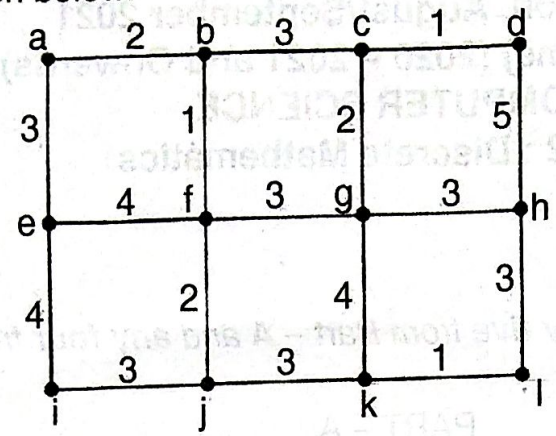
5. What is the expected value of the sum of the numbers that appear when a pair of fair dice is rolled ? **6**

6. a) A sequence of 10 bits is randomly generated. What is the probability that at least one of these bits is 0 ? **3**

   b) Let E_1 and E_2 be events in sample space S. Then show that $p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$. **3**

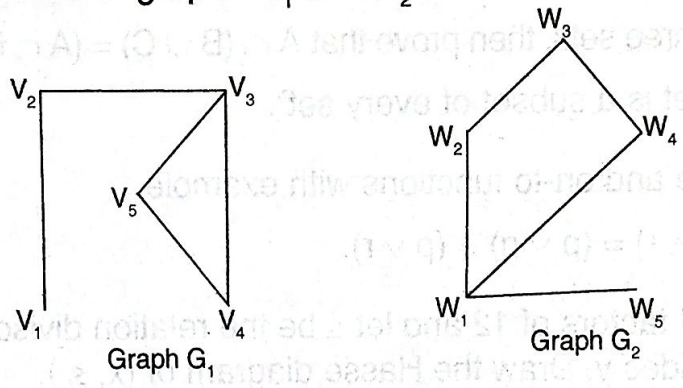
P.T.O.

7. Use Krushkal's algorithm to find a minimum spanning tree in the weighted graph given below. **6**



8. a) Define a bipartite graph. Show that the hypercube Q_3 is a bipartite graph. **3**

b) Check whether graphs G_1 and G_2 are isomorphic. **3**



PART – B

Answer **any four** questions. **Each** question carries **10** marks. **(4×10=40)**

9. a) Prove that $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg(q \vee p)$.

b) A survey among 100 students shows that of the three ice cream flavors Vanilla, Chocolate and Strawberry, 50 students like Vanilla, 43 like Chocolate, 28 like Strawberry, 13 like Vanilla and Chocolate, 11 like Chocolate and Strawberry, 12 like Strawberry and Vanilla and 5 like all of them. Find the number of students surveyed who like each of the following flavors.

i) Chocolate but not Strawberry.

ii) Chocolate and Strawberry but not Vanilla. **(5+5=10)**

10. a) By mathematical induction, prove that for every integer n, the number $A_n = 5^n + 2.3^{n-1} + 1$ is a multiple of B.

b) The matrix of a relation on the set A = {1, 2, 3} is given by $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$.

Show that R is an equivalence relation. **(5+5=10)**

11. a) Shirts numbered consecutively from 1 to 20 are worn by students of a class. When any 3 of these students are chosen to be debating team from the class, the sum of their shirt numbers is used as the code number of the team. Show that if any 8 of the 20 are selected, then from these 8 we may form at least two different teams having the same code number.

    b) A committee of eight people is formed from two mathematicians and ten economists. In how many ways can it be done, if the committee must include at least one mathematician ? **(5+5=10)**

12. a) State and prove the Bayes' theorem.

    b) The probability distribution of a discrete random variable is given below :

x	–2	–1	0	1	2	3
P(x)	0.1	K	0.2	2K	0.3	K

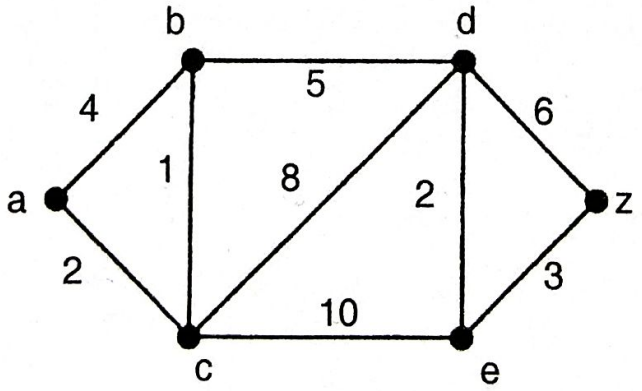
    Find : i) K ii) Mean and iii) Variance. **(5+5=10)**

13. a) Two cards are drawn from a pack of cards at random. What is the probability that it will be
        i) a diamond and a heart
        ii) a king and a queen
        iii) two kings ?

    b) State and prove the first theorem in Graph theory. Further, prove that the number of vertices of odd degree in a graph is always even. **(5+5=10)**

14. a) Define an Eulerian graph. If G is an Eulerian graph, then show that all the vertices of G are of even degree.

    b) Use Dijktra's algorithm, find the shortest path between a to z in the weighted graph.



(5+5=10)